GENERATOR-INVERTOR-DAMPER SYSTEM OF ELECTRON (POSITRON) BUNCHES MOVING IN COLD PLASMA FOR DEVELOPMENT OF STRONG ACCELERATING ELECTRIC FIELD

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overdense plasmas

Abstract

It is shown that high accelerating gradient can be obtained in a specially constructed system of electron (positron) bunches, moving in cold plasma with definite density. These combined bunch systems do not generate the wake fields behind them and can pass through the plasma column in a periodic sequence. The consideration is carried out numericaly and analyticaly in one dimensional approach, (which can be applied to finite system when its transverse dimensions are larger than plasma wave length, divided by 2π). The possibilities of the experimental tests by measuring the predicted energy gain are discussed on the examples of Argonne Wakefield Accelerator and induction linac with typical parameters.

1 INTRODUCTION

Langmuir already noticed, that the beam, which has passed the plasma column, contains a significant portion of electrons with the energies higher than the initial energy of the

beam. In the recent times various groups (see e.g. [1]-[3]) also observed experimentally the effect of the selfacceleration.

The possibility to accelerate the electrons of the bunches, moving in plasma, was considered numericaly in [4], [5] and some attempts of the analytical treatment of the problem have been performed in [6]- [9].

In the work [10], which is based on the previous results [11] used as a zero order approximation, nonlinear dynamics of the one-dimensional non rigid ultrarelativistic bunch of electrons, moving in the cold plasma, is considered by multiple scales perturbative approach. Square root of the inverse Lorentz factor of the bunch electrons is taken as a small parameter.

It was shown [10], in particular, that for specialy constructed system consisted from two bunches, first one dense enough $n_b^{(1)} \gg \frac{1}{2}n_0$, and the second one with low density $n_b^{(1)} \ll \frac{1}{2}n_0$, the electrons on the rare side of the second bunch, where electric field E is negative and large, can be significantly accelerated. In the first approximation of multiple scales method, the change of the momentum of the bunches electrons is given by $\triangle p_b = -eEt$, where t is a acceleration (deacceleration) time interval which, due to applicability of the multiple scales method, is $t \leq \omega_p^{-1}\gamma^{1/2}$.

The essential restriction on the acceleration rate in the considered cases comes from the steady state assumption adopted in [10]. If plasma electrons momenta in the wake field behind the first dense relativistic bunch, moving in underdense plasma, exciedes same critical value, allowed by steady state assumption, the plasma electron density behind the bunch turn out to be negative, which probably means that wake wave breaking limit is achieved. Restriction on the plasma electrons momenta inside the bunch automatically restricts the maximum value of the generated electric field.

In the present work another proper combination of the charged particles ($e^{\pm}$) bunches is proposed, which is constructed in such a way, that it does not generate the wake fields behind the combined bunch at all. The plasma electrons momenta, inside the combined bunch being negative, remains in the allowed domain and their maximum value is restricted only by the total momentum conservation low. The maximum of the

subsequent electric field then also can be large enough.

The combined bunch in the considered case consists (at least) of three parts:first one is a relativistic negative particles bunch, with density $n_b^{(1)} \gg \frac{1}{2}n_0$, which generates in a plasma inside the bunch strong electric field, which brakes the bunch particles (generator). The second bunch is a low density bunch with particles $n_b^{(2)} < \frac{1}{2}n_0$ (it can be negative particles bunch, positive particles bunch), which inverts the sign of the electric field generated by the first bunch (invertor). The third bunch with the parameters, which can coincide with that of the first one, provided on its rear part the zero values of electric field and plasma electrons momenta, coincided with the same values on the front of the first bunch (damper). It means that plasma behind the system of bunches remains unperturbed and no wake field is generated.

Acceleration of the particles of the rear part of the second and of the third bunches, where electric field is negative, will take place. The goal of the following calculations is to estimate the maximum possible values of the acceleration parameter.

In the next section of the work the vivid approach to the problem, based on the numerical solutions of the subsequent nonlinear equations and their graphic representations on phase plane are presented. The third section devoted to the analytical treatment of the problem, which based on the exact solutions of the equations and provides the general expressions for the energy gain, acceleration rate and acceleration length, the lengths of the different parts of the combined bunch as a functions of charge densities and Lorentz-factor of the bunches. Finally, the numerical examples are presented, which consider the possibilities of the experimental observations of the predicted selfacceleration of charged particles, in particular, on Argonne Wakefield Accelerator (AWA) and induction linacs.

2 PHASE PLANE IMAGES OF THE COMBINED BUNCHES, MOVING IN COLD PLASMA

Consider a plane rigid electron bunch with the infinite transverse dimensions and length $[0, d]$ along the z-axis. Velocity of the bunch in the lab system is $v_z = v_0 \quad (\beta = \frac{v_0}{c})$, plasma is cold with the immobile ions. The charge density of the plasma electrons is

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n_b;the case, when $n_b > \frac{1}{2}n_0$ called underdense regime, $n_b < \frac{1}{2}n_0$-overdense regime. The plasma electrons motion equations described in hydrodynamic approximation is

$$\frac{\partial \rho_e}{\partial t'} + \beta_e \frac{\partial \rho_e}{\partial z'} = -E, \text{ or } (\beta_e - \beta_0)\frac{\mathrm{d}\rho_e}{\mathrm{d}\tilde{z}} = -\mathrm{E}; \tag{1}$$

Continuity eq. for plasma electrons is

$$\frac{\partial n_e}{\partial t'} + \frac{\partial}{\partial z'}(\beta_e n'_e) = 0, \text{ or } \frac{\mathrm{d}}{\mathrm{d}\tilde{z}}[\mathrm{n'}_e(\beta_e - \beta)] = 0, \tag{2}$$

and Maxwell (Coulomb) equation for the electric field is

$$\frac{dE'}{dz} = \frac{E'}{d\tilde{z}'} = n'_0 - n'_e - n'_b \tag{3}$$

In eqs. (1-3)

$$t' = \omega_p t \quad z' = k_p z, \omega_p = \left(\frac{4\pi e^2 n_b}{m}\right)^{1/2},$$

$$E = (4\pi nmc^2)^{1/2}E' = \frac{mc\omega_p}{e}E', n'_e = \frac{n_e}{n}, n'_b = \frac{n_b}{n}$$

$$\rho_e = \frac{\rho_{ez}}{mc}, \beta_e = \frac{v_{ez}}{c}, \gamma = (1-\beta^2)^{-1/2};$$

$\tilde{z} = z - v_0 t$ (steady state approximation, n- is an arbitrary density, which finaly will be chosen).

Continuity eq. (2) has an integral

$$n_e(\beta - \beta_e) = n_0\beta; \tag{4}$$

constant is defined from the boundary condition, $z = d, \beta_e = 0, n_e = n_0$. In (4) and in what follows the superscripts "prime" are omitted.

From eq. (1) it follows that

$$E = -\frac{d\Phi}{d\tilde{z}} \equiv -\Phi'; \Phi \equiv \sqrt{1 + \rho_e^2} - \beta\rho_e \tag{5}$$

and eqs (2)-(3) can be written in the form

$$\Phi'' - \beta\gamma^2 \frac{\Phi}{(\Phi^2 - \gamma^{-2})^{1/2}} = \alpha - \gamma^2 \tag{6}$$

(see also [11], [13]-[14]).

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Eq. (6) has an "energy" integral

$$\mathcal{E} = \frac{1}{2}\Phi'^2 + \gamma^2\left[\Phi - \beta(\Phi^2 - \gamma^{-2})\right] - \alpha\Phi \qquad (7)$$

where $\alpha \equiv n_b/n_0$;the boundary conditions at $\tilde{z} = d$ are $\rho_e = 0, \Phi = 1, \Phi' = 0$ and gives $\mathcal{E} = 1 - \alpha$, but in what follows the eq. (7) considered at the arbitrary boundary conditions.

The integral (7) allowed to interpret equations (1-3) as equations for the point with unit mass with the "coordinate" Φ and "velocity" Φ' moving in the potential $U = U_0 + U_1 = \gamma^2\left[\Phi - \beta\left(\Phi^2 - \gamma^{-2}\right)^{1/2}\right] - \alpha\Phi$.

On Fig. 1 the U as a function of Φ is drawn for different values of α and for $\gamma = 10$. The negative values of $\alpha = \frac{n_b}{n_0}$ correspond to the bunch of positive charged particles, positive α corresponds to the bunch of negative charged particles. For $\alpha < 0, \gamma^{-1} \leq \Phi \leq 1$ and $\Phi \geq 1$for $\alpha > 0$, when boundary conditions, when $\bar{z} = d$, are $\Phi = 1$ and $\Phi' = 0$. From Fig. 1 it is seen that for $\alpha < 0$ the motion is always periodic, it is also periodic for $0 \leq \alpha \leq 1/2$, and non periodic for $\alpha > 1/2$ [11].
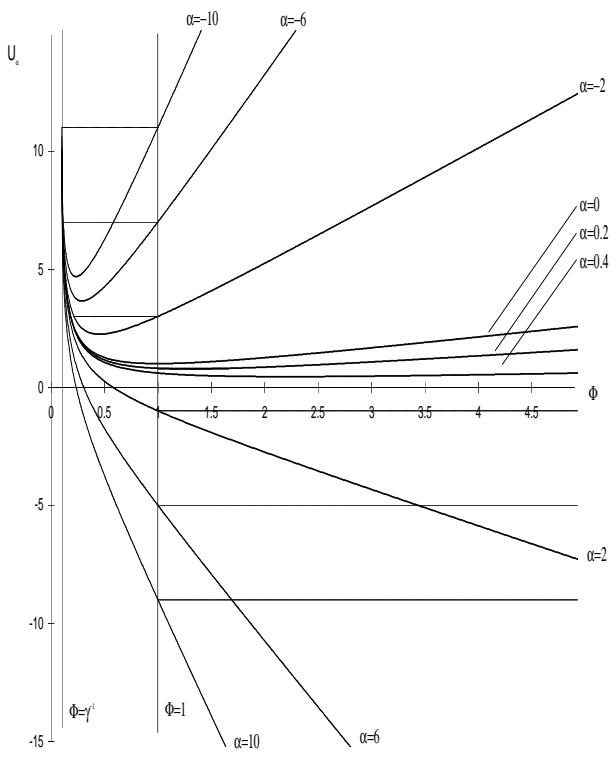
Fig.1. Curves of potential Uα(Φ) for different α.

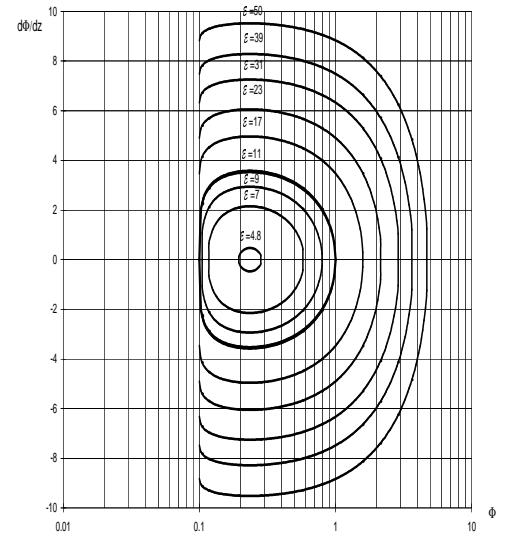α=−10 α=−6 α=−2 α=0 α=0.2 α=0.4 α=2 α=6 α=10

U$_\alpha$ Φ

Φ=γ⁻¹ Φ=1

Fig. 2. Phase-plane portrait for α=-10, γ=10 for different energies $\mathcal{E}$.
$\mathcal{E}$ =11 is the separatrix.

dΦ/dz Φ

$\mathcal{E}$ =60 $\mathcal{E}$ =39 $\mathcal{E}$ =31 $\mathcal{E}$ =23 $\mathcal{E}$ =17 $\mathcal{E}$ =11 $\mathcal{E}$ =9 $\mathcal{E}$ =7 $\mathcal{E}$ =4.8
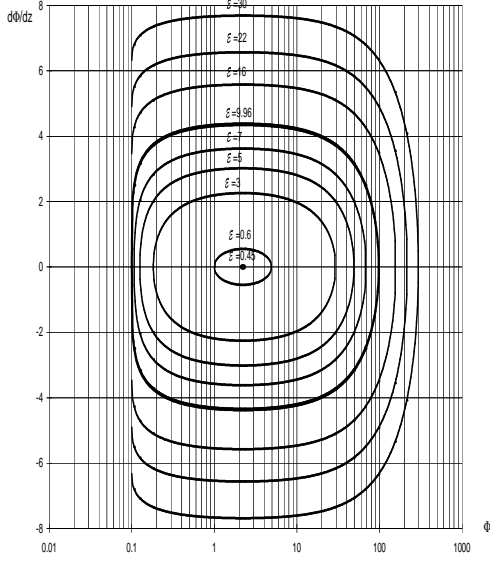
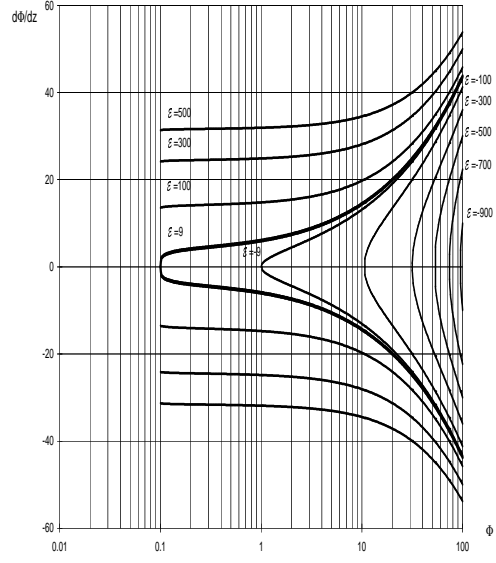Fig. 3. Phase-plane portrait for α=0.4, γ=10. Separatrix: $\mathcal{E}$ =9.96 .

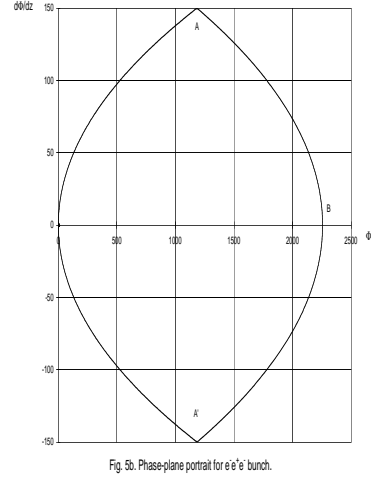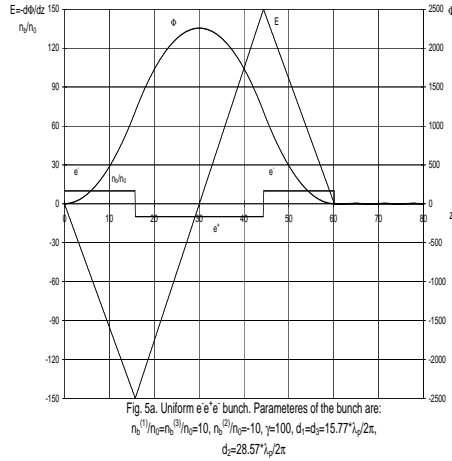Fig. 4. Phase-plane portrait for α=10, γ=10. Separatrix: $\mathcal{E}$ =9.

Fig. 2, 3, 4 represent the phase plane (Φ', Φ) portrait of the system of cold plasma-rigid charged particle bunch in the cases, when $\alpha = -10, 0.4, 10$ and $\gamma = 10$ at different boundary conditions (different $\mathcal{E}$). The closed phase trajectories correspond to periodic motion.

Trajectories on Fig. 2, 3, 4 can serve as a building blocks to construct the phase portraits of the combined bunches, moving in cold plasma. It must be represented by a closed loop, starts from the boundary condition $\Phi = 1, \Phi' = 0$ and accomplished at the same point $\Phi = 1, \Phi' = 0$ on the phase plane. An example of such a loop is presented on Fig. 5. The loop on Fig. 5b from the point $0 :\quad \Phi = 1, \Phi' = 0 \quad (\rho_e = 0, v_e = 0, E = 0,$ when $\tilde{z} = d)$ and the curve OA represents the motion of the bunch of negative charged particles (generator) $\alpha^{(1)} = 10$, when at the rear side of the bunch very strong but positive electric field $E_0 = -\Phi'$ can be generated. Taking this point as a new boundary conditions for the second bunch of positive charged particles (invertor), the trajectory is turned to the point B, where electric field is zero and then to the point A' where the field $-E_0$ is obtained. From point A' to the point 0 the trajectory described the motion of the third

bunch of the negatively charged particles (damper) on the rear side of which the initial boundary conditions exist. It means that no wake field excites and the plasma behind the third bunch remains unperturbed. As a consequence, the above mentioned restriction on the value of the field E_0, which arouse from the condition of the stability of the wake field, is removed.

Fig. 5a. Uniform e⁻e⁺e⁻ bunch. Parameteres of the bunch are:
$n_b^{(1)}/n_0 = n_b^{(3)}/n_0 = 10$, $n_b^{(2)}/n_0 = -10$, $\gamma = 100$, $d_1 = d_3 = 15.77^* \lambda_p/2\pi$,
$d_2 = 28.57^* \lambda_p/2\pi$

Fig. 5b. Phase-plane portrait for e⁻e⁺e⁻ bunch.

This is not only the condition for the applicability of steady state regime for the description of the zero order approximation in the multiple scales approach, but also open the possibility to repeat more than once the procedure, sending the next combined bunches into the unperturbed by previous bunches plasma.

The Lorentz factor of all three bunches in the presented case is, of course, the same $\gamma = 100$, and
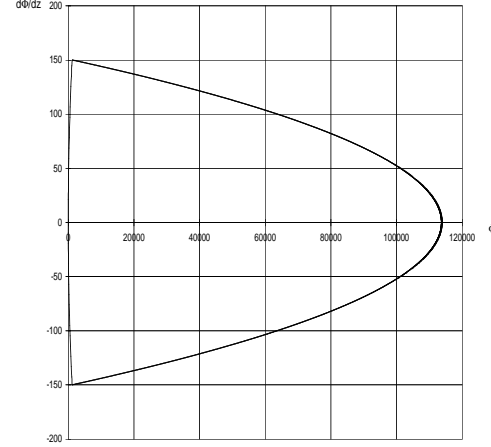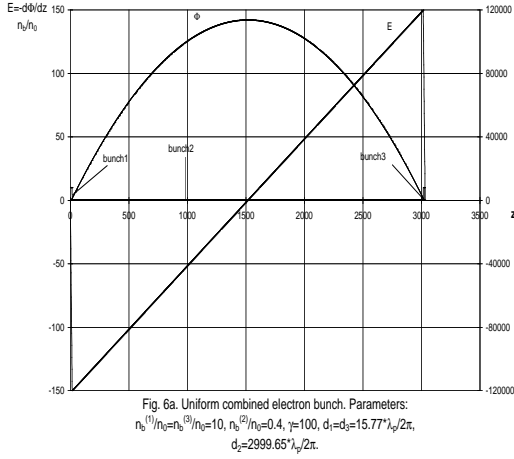
$$\alpha^{(2)} = -\frac{n_b^{(2)}}{n_0} = -10, \alpha^{(3)} = \alpha^{(1)} = \frac{n_b^{(3)}}{n_0} = 10.$$

The total length of the positron bunch is $28.57\frac{\lambda_p}{2\pi}$ and lengths of the first and third electron bunches are $\sim 15.77\frac{\lambda_p}{2\pi}$ each.

On Fig. 5a the potential Φ and electric field $E = -\frac{\partial \Phi}{\partial \tilde{z}}$ as a function of $\tilde{z}$ are presented. It is evident from Fig. 5a that positrons of the first part of the second bunch, where $E > 0$, can be accelerated as well as all electrons of the third bunch ($E < 0$).

The similar picture may be obtained by construction of the combination of three bunches with negatively charged particles (electrons). First bunch is dense $n_b^{(1)}/n_0 \gg 1$,

second has a low density $\frac{1}{2}n_0 - n_b^{(2)} \gg \gamma^{-2}$, and third is again dense $n_b^{(3)} \gg n_0$.

Fig. 6a. Uniform combined electron bunch. Parameters:
$n_b^{(1)}/n_0 = n_b^{(3)}/n_0 = 10$, $n_b^{(2)}/n_0 = 0.4$, $\gamma = 100$, $d_1 = d_3 = 15.77 \cdot \lambda_p/2\pi$,
$d_2 = 2999.65 \cdot \lambda_p/2\pi$.

Fig. 6b. Phase-plane portrait for combined electron bunch.

The example of such a combination is presented on Fig. 6a, 6b, where electric field and phase portrait are drawn for the case, when

$$n_b^{(1)} = n_b^{(3)} = 10n_0, n_b^{(2)} = 0.4n_0, \gamma = 100.$$

It is seen from Fig. 6a, that the length d_2 of the second bunch-invertor is too large in considered case. Acceleration of the bunches electrons take place when $E < 0$ i.e. mostly in the rear side of the second bunch-invertor and, at some extend, in the third bunch-damper.

The role of the bunch-invertor may play also the plasma itself $(n_b^{(2)} = 0)$. In such a case the electric field interacts on plasma electrons, which are moving in the direction opposite to that of bunch, with the initial velocity equal to zero. So they will accelerate in the region where $E > 0$, and drag in the region where $E < 0$. The electrons of the third bunch, where $E > 0$ will accelerate. The charge distribution inside the bunches can be nonuniform.

On Figs. 7a, 7b the results of the numerical solution of eqs. (1-3) are displayed for the case of sinusoidal charge distribution inside the electron- positron-electron bunches span by plasma in-between. The bunches have the equal lengths $d = 15\frac{\lambda_p}{2\pi}$, devided by

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plasma columns with lengths $d_0 = 60\frac{\lambda_p}{2\pi}, \gamma = 100,$

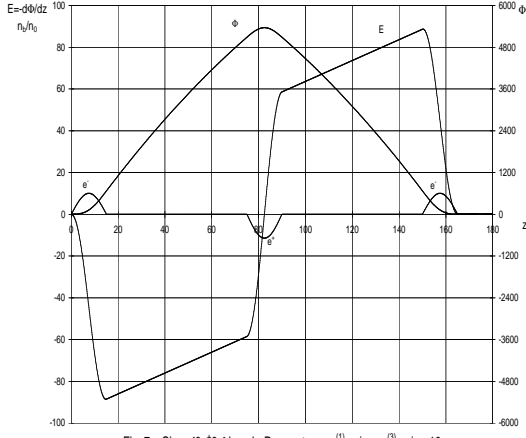$$\frac{n_b^{(1)}}{n_0} = \frac{n_b^{(3)}}{n_0} = 10, \frac{n_b^{(2)}}{n_0} = -11.48.$$

Fig. 7a. Sine e⁻0e⁺0e⁻ bunch. Parameters: $n_b^{(1)}{}_{max}/n_0 = n_b^{(3)}{}_{max}/n_0 = 10$, $n_b^{(2)}{}_{max}/n_0 = -11.4831$, $\gamma = 100$, $d_1 = d_2 = d_3 = 15*\lambda_p/2\pi$, $d_0^{(1)} = d_0^{(2)} = 60*\lambda_p/2\pi$.
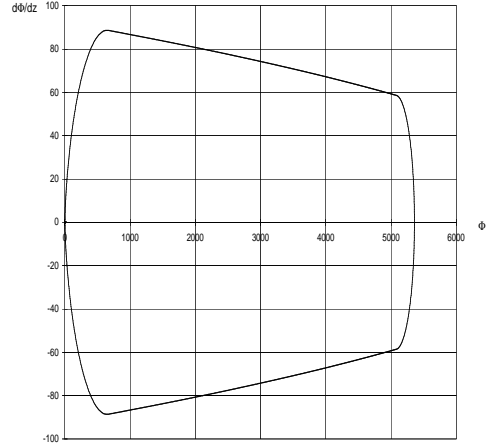
Fig. 7b. Phase-plane portrait for sine e⁻0e⁺0e⁻ bunch.

It is neccessary to note that the wake field behind the combined bunch due to small in-accuracies, presented in actual calculations practically exists, but the amplitude of this wake is small and far from wave breacking limit. This last condition defines the tolerances on the parameters of combined bunch and must be taken into account in actual calcula-tions for the proposed experimental tests. In the case of the sequence of the combined bunches, passing through plasma, the conditions, which define the tolerances on bunch parameters, will be more severe.

3 ANALYTICAL APPROACH

In this section the results of the exact analytical solutions of eqs. (1-3) for the com-bined bunch with the uniform charge distributions inside the constituting bunches are presented. Results for single rigid (electron) bunch with the zero boundary conditions $\rho_e = 0, E = 0$, when $\tilde{z} = d$) are obtained in [11].

Boundary conditions at the end of the first electron bunch moving in underdense

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plasma $n_b \gg \frac{1}{2}n_0$ are $\rho_e = \rho_0, E = E_0$, where

$$E_0 = \pm \left[2\left(n_b^{(1)} - n_0\right)\right]^{1/2} \left\{(1 + \rho_0^2)^{1/2} - 1 - \alpha^{(1)}\beta\rho_0\right\}^{1/2} \tag{8}$$

$$\alpha^{(1)} \equiv \qquad n_b^{(1)}/n_b^{(1)} - n_0$$

The field inside the second electron bunch, moving in overdense regime $n_b^{(2)} < 1/2n_0$ with the same velocity is

$$E^{(2)} = \pm \left[2\left(n_0 - n_b^{(2)}\right)\right]^{1/2} \left\{a - (1 + \rho_e^2) - \alpha^{(2)}\beta\rho_e\right\}^{1/2},$$

$$\alpha^{(2)} = \qquad \frac{n_b^{(2)}}{n_0 - n_b^{(2)}} < 1, \quad \alpha^{(2)}\beta < 1,$$

$$a \equiv \qquad 1 - \frac{n_b^{(1)} - n_b^{(2)}}{n_0 - n_b^{(2)}} \left\{\left[1 - (1 + \rho_0^2)^{1/2}\right] - \beta\rho_0\right\}; \tag{9}$$

When $\rho_0 \to 0 \quad a \to 1$, when

$$\rho_0 < 0, |\rho_0| \gg 1$$

$$a \approx \frac{n_b^{(1)} - n_b^{(2)}}{n_0 - n_b(2)}(1 + \beta)|\rho_0| \gg 1 \tag{10}$$

When plasma electron momenta is equal to

$$\rho_\pm = -\frac{a\alpha^{(2)}\beta}{1 - (\alpha^{(2)}\beta)^2} \pm \left[\frac{\left(a\alpha^{(2)}\beta\right)^2}{1 - (\alpha^{(2)}\beta)^2} + \frac{a^2 - 1}{1 - \alpha^{(2)}\beta}\right]^{1/2} \tag{11}$$

the $E^{(2)} = 0$; when $|\rho_0| \gg 1$ and $a \gg 1$

$$\rho_- = -\frac{a}{1 - \alpha^{(2)}\beta}, |\rho_-| > |\rho_0| \tag{12}$$

Note that it is impossible to obtain $E = 0$ and $\rho_e = 0$ simultaneously using only the second (electron, $n_b^{(2)} > 0$, or positron, $n_b^{(2)} > 0$) bunch.

The second root ρ_+ will not be reached in the considered cases.

The electric field $E^{(2)}$ has a maximum at

$$\rho_e^{max} = -\frac{\alpha^{(2)}\beta}{[1 - (\alpha^{(2)}\beta)^2]^{1/2}} \tag{13}$$

In the considered cases ρ_e^{max} is choosen according the condition

$$|\rho_-| > |\rho_0| > |\rho_e^{max}|.$$

The length of the second bunch is choosen in such a way that at the end of it $E^{(2)} = -E_0$, and $\rho_e = \rho_0$ (see Fig. 6).

The third bunch then can be similar to the first one in order to dump the electric field and plasma electron momenta to their initial zero values, in order to dump wake fields completely. For this end the length $d^{(2)}$ must be found, integrating the equation

$$dz = \frac{\pm[\beta(1 + \rho_e^2) - \rho_e]d\rho_e}{[2(n_0 - n_b^{(2)})]^{1/2}(1 + \rho_e^2)^{1/2}[a - (1 + \rho_e^2)^{1/2} - \alpha^{(2)}\beta\rho_e]^{1/2}} \tag{14}$$

by two steps-from ρ_0 to ρ_- and from ρ_- to ρ_0, taking sign $+$ when ρ_e increases with the $\tilde{z}$ and sign $-$, when ρ_e decreases with the z. In both cases $|\rho_e| \geq \rho_0 \gg 1$ and the integration simplifies essentially, when $(1 + \rho_e^2)^{1/2}$ replaced by $|\rho_e|$;the result of integration, using (10, 12), is

$$d_2^e = \frac{4(1 + \beta)|\rho_0|^{1/2}}{[2(n_0 - n_b^{(2)})]^{1/2}(1 - (\alpha^{(2)}\beta))^{1/2}} \left[\frac{(n_b^{(1)} - n_b^{(2)})}{(n_0 - n_b^{(2)})}\frac{(1 + \beta)}{(1 - \alpha^{(2)}\beta)} - 1\right]^{1/2} \tag{15}$$

In the case, when the second bunch is consisted from positive particles (positrons) the length of the second bunch, obtained by the similar calculations, is

$$d_2^p = \frac{4(1 + \beta)|\rho_0|^{1/2}}{[2(n_0 + \bar{n}_b^{(2)})]^{1/2}(1 + \bar{\alpha}^{(2)}\beta)^{1/2}} \left[\frac{(n_b^{(1)} + \bar{n}_b^{(2)})}{(n_0 + \bar{n}_b^{(2)})}\frac{(1 + \beta)}{(1 + \bar{\alpha}^{(2)}\beta)} - 1\right]^{1/2} \tag{16}$$

where $\bar{\alpha}^{(2)} \equiv \frac{\bar{n}_b^{(2)}}{n_0 + \bar{n}_b^{(2)}}$

In the positron case $n_b^{(2)}$ can be choosen as $\bar{n}_b^{(2)} = n_b^{(1)} \gg n_0$ for the effective inversion of the electric field E_0. Comparison of the eqs. (15) and (16) for

$$n_b^{(2)} \ll n_0, n^{(1)} \gg n_0 \quad \bar{n}_b^{(2)} = n_0, \beta = 1,$$

shows, that in this case

$$\frac{d_2^p}{d_2^e} = \frac{n_0}{2n_b^{(1)}} \ll 1 \tag{17}$$

i.e. the dense $\bar{n}_b^{(2)} = n_0$ positron bunch-invertor would be much shorter than electron bunch with $n_b^{(2)} < 1/2n_0$. The length of the first (and third) electron bunch can be obtained similiarly or from the results of work [12] for the case, when

$$1 \ll \frac{n_b^{(1)}}{n_0} \ll 2\gamma^2, |\rho_0| \gg 1, \beta \to 1,$$

and is

$$d_1 = \beta |\rho_0|^{1/2} = d_3 \tag{18}$$

The possible largest plasma electron momenta inside the first bunch in the considered case differs from that found in [11] by wake wave breaking limit and can be estimated from momentum conservation (see also [10]),

$$n_b^{(1)} \beta \gamma d^{(1)} \geq \int_0^{d_1} \rho_e(\tilde{z}) n_e(\tilde{z}) dz = \frac{n_0 |\rho_0| d_1}{4} \tag{19}$$

For the estimate in (19) the expression for n_e, obtained from continuity equation

$$n_e(\tilde{z}) = \frac{n_0 \beta (1 + \rho_e)^{1/2}}{\beta (1 + \rho_e^2)^{1/2} - \rho_e},$$

is used, which approximated by linear one in interval from $n_e = n_0$ at $p_e = 0$ up to the value $n_e = \frac{n_0 \beta}{1 + \beta}$ for large ρ_e.

From (19)

$$1 \ll |\rho_0| \leq \frac{4 n_b^{(1)}}{n_0} \gamma, \tag{20}$$

and corresponding electric field from (8) and (20) is

$$|E_0| = 2 \left(n_b^{(1)} \right)^{1/2} |\rho_0|^{1/2} \leq \frac{4 n_b^{(1)}}{n_0^{1/2}} \gamma^{1/2} \tag{21}$$

As it was shown in [10] the change of the bunch electron momenta $\triangle p$, in the first approximation, which can be found from the second order equations, is

$$\triangle p = p_{b1} = -E(\tilde{z}) \tau \tag{22}$$

where $E(\tilde{z})$ is the electric field in the zero order approximation inside the rigid bunch and $\tau = \omega_p t \leq \gamma^{1/2}$. The limit on acceleration time interval follows from the domain of validity of the applied multiple scales approach.

The length of the plasma column needed for the change of the momenta given by (22) is, in ordinary units

$$l = ct \leq \frac{c}{\omega_p} \gamma^{1/2} = \frac{\lambda_p}{2\pi} \gamma^{1/2} \tag{23}$$

The acceleration rate (gradient), in ordinary units is

$$G = \frac{c \triangle p}{el} \tag{24}$$

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4 POSSIBILITIES OF THE EXPERIMENTAL TESTS

The formulae obtained in sec. 3 for the parameters of proposed device in ordinary units and $n = n_0$ are from (20):

$$|\rho_0| \leq \frac{4n_b^{(1)}}{n_0}\gamma mc, \tag{25}$$

from (21)

$$E_0 \leq 4\left(\frac{n_b^{(1)}}{n_0}\right)\gamma^{1/2}\frac{mc\omega_p}{e}, \tag{26}$$

from (22), (26)

$$\triangle p_b = eE_0 t \leq 4\frac{n_b^{(1)}}{n_0}mc\gamma = \frac{4n_b^{(1)}}{n_0}p_b; \tag{27}$$

$$\frac{\triangle p_b}{p_b} \leq 4\left(\frac{n_b^{(1)}}{n_0}\right); \quad t \leq \omega_p^{-1}\gamma^{1/2}$$

.

The accelerating gradient from (24), (27) is

$$G = \frac{c\triangle p_b}{el} = \frac{\triangle p}{et} \sim 4\left(\frac{n_b^{(1)}}{n_0}\right)\frac{mc\gamma^{1/2}\omega_p}{e} \tag{28}$$

$$G \sim \frac{4\pi}{\lambda_p}\left(\frac{n_b^{(1)}}{n_0}\right)\gamma^{1/2}MV/cm,$$

$$\lambda_p = \frac{2\pi c}{\omega_p} = 6.08 \cdot 10^{-4}\left(\frac{3\cdot 10^{19}}{n_0}\right)^{1/2}cm$$

The acceleration length (22):

$$l = ct \leq \frac{\lambda_p}{2\pi}\gamma^{1/2}.$$

The lengths of the first and third electron bunches (18), (25) are:

$$d_1 = d_3 \leq \frac{\lambda_p}{\pi}\gamma^{1/2}\left(\frac{n_b^{(1)}}{n_0}\right)^{1/2} \tag{29}$$

The length of the second electron bunch (15), (25) is

$$d_2^e \leq \frac{8\lambda_p}{\pi}\gamma^{1/2}\left(\frac{n_b^{(1)}}{n_0}\right) \tag{30}$$

$$n_b^{(2)} \ll n_0 \ll n_b^{(1)}$$

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The length of the second positron bunch (16), (25) is:

$$d_2^p \leq \quad \frac{4\lambda_p}{\pi}\gamma^{1/2} \tag{31}$$

$$\bar{n}_b^{(2)} = \quad n_b^{(1)} \gg n_0;$$

According to (26) it is possible to obtain essential acceleration fields when the previously accelerated dense bunches of negative charged particles (electrons) are used. The lengths of such a bunches, according to (29), imcreases as $\sim \gamma^{1/2}$, so it seems that they must be the bunches from high current accelerators with the energies up to tens of Mev. Even at these energies maximal acceleration gradient is high enough, it can exceed the acceleration gradient of the ordinary linacs by a several orders of magnitude.

Consider the experimental possibilities of proof-of-principle experiment on existing accelerators or test facilities. It seems that Argonne Wakefield Accelerators (AWA), which accelerates very dense bunches, is suitable for above mentioned proposes.

AWA [17] is $20MeV$ electron accelerator which accelerate the bunches of $20p$ sec full width (the length $d_{1exp} = 0.6cm$), diameter about $0.1cm$ [18], the total charge of the bunch 100nC, i.e. $n_b^{(1)} = 2 \cdot 10^{14} cm^{-3}$. If n_0 is choosen as $n_0 = \frac{1}{5}n_b^{(1)} = 4 \cdot 10^{13} cm^{-3}$, then the optimal length of the first bunch from (29) is $d_1 = 2.4cm$, i.e. 4 times larger than the experimentally existing one $d_{1exp} = 0.6cm$.

The role of the invertor can serve the plasma column itself. Witness bunch, which has 10ps duration, less than 1nC total charge, and energy 4Mev can be used as accelerated bunch, being placed somewhere between first and third bunches. Third bunch (damper) has similiar parameters as the first one and must be placed at distance $d_2 = 8\left(\frac{n_b^{(1)}}{n_0}\right)^{1/2} d_1 \approx 43cm$ from the end of the first bunch. Experimentally it is possible to put it on the distance $d_{2exp} = 46cm$, because the frequency of the RF-cavities is $\nu = 1.3GHz \quad (d_{2exp} = nc/\nu = n \cdot 23cm, n = 1, 2, 3, \cdots)$. If the length of the first bunch could be made 4 times large than existing one and total charge of the bunch will be also 4 times larger-400nC, then the change of the energy of the witness bunch will be, according to (27)

$$\triangle \mathcal{E}_w \leq 4\frac{n_b^{(1)}}{n_0}\mathcal{E}_1 = 400MeV \gg \mathcal{E}_w = 4\text{MeV}.$$

For the existing experimental parameters of AWA:$d_1 = 0, 5cm, d_2 = 23cm$, total charge of bunch 70nC, $n_b^{(1)} = 1.4 \cdot 10^{14} cm^{-3}$ results of the numerical calculations, along the lines presented in Sec. 2, are shown on figures 8a and 8b. The role of bunch-invertor plays the plasma column, with density n_0 as a free parameter, which was been chosen from the necessity to invert the field, produced by the first bunch, in such a way, that the succesive bunch, with the same parameters as the first one, moving at fixed (by RF-frequency) distance from the first one, $d_2 = 23cm$, will be able to dump the field to zero. The sought density of plasma n_0 was found equal to $n_0 \approx 1.17 \cdot 10^{13} cm^{-3}$, $\lambda_p = 0.973cm$, $n_b/n_0 = 11.95$. Then the maximum value of the accelerated electric field is equal to $E_{max} \approx 100 Mv/cm$, and $E < 0$ on the rear part of the second (third) bunch. So about the half of the electrons of the second (third) bunch, as it can be seen from Fig. 8a, can be accelerated during the time $t \sim \omega_p^{-1} \gamma^{1/2}$ up to energy $\mathcal{E}_f \approx 70 MeV$.
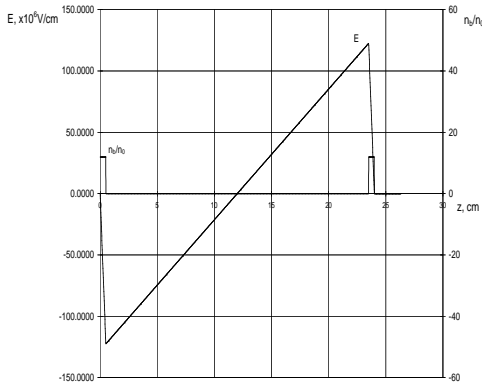

Fig. 8a. Electric field E and n_b/n_0 for proposed experiment on AWA. Bunch und plasma parameters are: $n_b^{(1)} = n_b^{(3)} = 1.4 \times 10^{14} cm^{-3}$, $n_b^{(2)} = 0$, $n_0 = 1.1714 \times 10^{13} cm^{-3}$, $d_1 = d_3 = 0.5cm$, $d_2 = 23cm$, $\gamma = 40$, $\lambda_p = 0.973cm$.
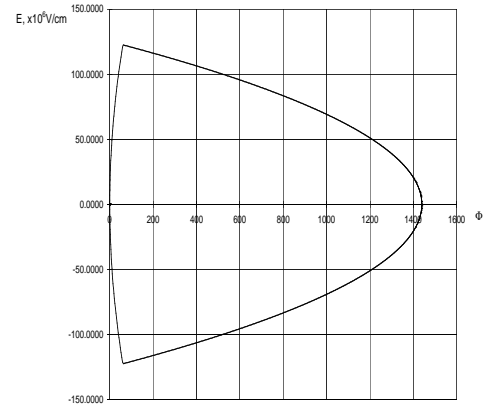

Fig. 8b. Phase-plane portrait for proposed experiment on AWA.

Proposed experiment can serve as a prof-of-principle one;plasma remains practically unperturbed after the passage of the first and the second bunches, so it is possible to inject the next couple of the bunches and also obtain acceleration, again leaving the plasma behind the bunches in practically unperturbed state and so on. As it was above mentioned in order to obtain the optimal experimental conditions, it is necessary to enlarge the micropulse duration, leaving its charge density the same. This will rise the maximum

obtainable electric field and energy gain. Of course, the obtained numbers for E_{max} and $\mathcal{E}$ are at some extent tentative;it is neccessary to remember that they are obtained in the model of the bunch with the infinite transverse dimensions and uniform charge distribution. More detailed calculations must be performed in the course of preparation of the experiment.

However, taking into account the results of the numerical calculation for laser driven wake fields in generalized vortex-free plasma [19], it is possible to adopt the statement of authors [19], that "even when driver's width approaches c/ω_p the 1-D (nonlinear theory) predictions are still a good guide for determining the accelerating field strength". In the considered case of AWA the diameter of the bunch is $D = 0.1cm \leq \frac{c}{\omega_p} = \frac{\lambda_p}{2\pi} = 0.15cm$, so for the existing parameters of AWA predictions of 1-D nonlinear theory can be valid.

For the improved parameters of AWA, as it was shown, $n_0 = 4 \cdot 10^{13}cm^{-3}$ and $D = 0.1cm \geq \frac{c}{\omega_p} = \frac{\lambda_p}{2\pi} = 0.08cm$ and the situation is more favourable.

For Accelerator Test Facility (ATF) at BNL the situation in the considered respect is worse, due to the very narrow bunches $r = 0.03cm$ and smaller bunch charge $Q = 2nC$ [20]. At any case, the considered problem needs the development of 2-D nonlinear theory of wake field generation by the electron (positron) bunches with the finite transverse dimension.

Another possibility to check the predicted effect is based on the use of induction linacs. Consider the example of the accelerator with the parameters, which are within the range of the possibilities of the existing or planned induction linacs (see e.g. [15], [16]): Energy $\mathcal{E} = 16MeV$

Pulse current $\quad I = 5kA$

Pulse duration $\quad \tau = 2.5nsec$

Diameter of the bunch $D = 2cm$

Then the charge density of the first and third bunches will be $n_b^{(1)} = 1.6 \cdot 10^{11}cm^{-3}$, the plasma density can be chosen as $n_0 = \frac{1}{5}n_b^{(1)}$ and then, according to (29, $d_1 \approx 75cm$, the distance between second and third bunches, according to (30), is $d_2 \approx 13.4m$ (the plasma column plays the role of invertor). The maximum energy gain of the witness bunch of

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much smaller current (e.g. 0.1 I and $\tau < 2.5 nsec$), placed in the proper position between the first and third bunches, according to (27), is

$$\triangle \mathcal{E}^{max} = 4 \cdot 5 \cdot 16 = 320 MeV.$$

The condition of the applicability of the 1-D nonlinear thery in this case is practically fulfiled:

$$D = 2cm \leq \frac{\lambda_p}{2\pi} = 2.3cm.$$

If the positron bunch can be used as a second invertor bunch with the same current I as the first one, then the length d_2, according to (31), can be essentially reduced- $d_2^p = 134.1cm$, and the part of positrons from this bunch can be accelerated, up to the energies $\approx 300 MeV$.

It is evident, that in all cases considered, the length of the plasma column must be 2-3 times larger than the total length of the combined bunch, due to transient effects.

The increase of the lengths of the bunches with the energy may be considered as tolerable one, when the extended astrophysical objects, as e.g. supernovae, are considered. Then proposed mechanism can be applied, for example, even for the explanation of the origin of the superhigh energy cosmic rays. This problem (see e.g. [21]) is now the most important one for the astrophysics of cosmic rays with the energies $\mathcal{E} > 10^{17} eV$, when all known mechanisms of cosmic rays acceleration are uneffective [22].

In order to illustrate this possibility consider the following numerical example. Suppose that the flux of electrons with the initial energy $\mathcal{E}_i = 5 \cdot 10^{16} eV$ already obtained by some effective and commonly adopted mechanism. The flux with the density $n_b^{(1)} = 1.5 \cdot 10^{14} cm^{-3}$ passes through the electron-proton plasma with the density $n_0 = 3 \cdot 10^{11} cm^{-3}$. Then it followed by positron (2) and electron (3) fluxes with the same densities $n_b^{(2)} = n_b^{(3)} = n_b^{(1)}$. Then some part of positron and third-electron bunches can be accelerated up to final energy

$$\mathcal{E}_f^{max} = 4 \left(\frac{n_b^{(1)}}{n_0} \right) \mathcal{E}_i = 10^{20} eV$$

i.e. reaches the atmost limit of the observed cosmic rays. The length of the electron fluxes is 140km, the length of the positron bunch is 25km. (Let us mention that up to now observed events at such energies are very rare). Considered example is only illustrative and aplicability of the proposed mechanism to the problem of the origin of the superhigh energy cosmic rays needs, of course, more careful consideration.

In addition, it is necessary to mention,that the expression for momentum change (27) $\triangle p = -eEt$, obtained for the first approximation of the developed multiple scales method,is valid up to fifth approximation:

$$\frac{p_b}{mc} \equiv \rho_b = \rho_{b0} + \epsilon \rho_{b1} + \epsilon^5 \rho_{b5} + \epsilon^6 \rho_{b6} + \cdots, \quad \epsilon = \gamma_0^{-1/2}, \tag{32}$$

as it follows from the results of [10]. Hence it is possible to assume,that eq. (27) is valid for the time intervals larger than $\omega_p t \leq \epsilon^{-1} = \gamma^{1/2}$ used in the present work for estimites. But in order to proof this assumption it is necessary to develop more complicated version of multiple scales method, than used in [10], where only three scales are introduced:

$$\tilde{z} = z - \beta_0 t, \quad \tau_1 = \epsilon t, \quad \zeta_1 = \epsilon z.$$

Enlargement of the validity domain on t needs consideration of the contributions from the next scales

$$\tau_2 = \epsilon^2 t, \quad \tau_3 = \epsilon^3 t, \quad \tau_4 = \epsilon^4 t$$

to the momentum development (32).

5 AKNOWLEDGMENTS

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